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# ROHM & HAAS COMPANY

## REDSTONE ARSENAL RESEARCH DIVISION

## HUNTSVILLE, ALABAMA

Report No. S-45

THE STUDY OF THE MECHANISM OF ENHANCED

BURNING RATE OF SOLID PROPELLANTS( U)

ORDNANCE CORPS, DEPARTMENT OF THE ARMY

FOR ERRATA

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## E R R A T A

The attached figures should be inserted in your copy of
Rohm & Haas Company Report S-45 "The Study of the Mechanism
of Enhanced Burning Rates of Solid Propellants ( U) ".

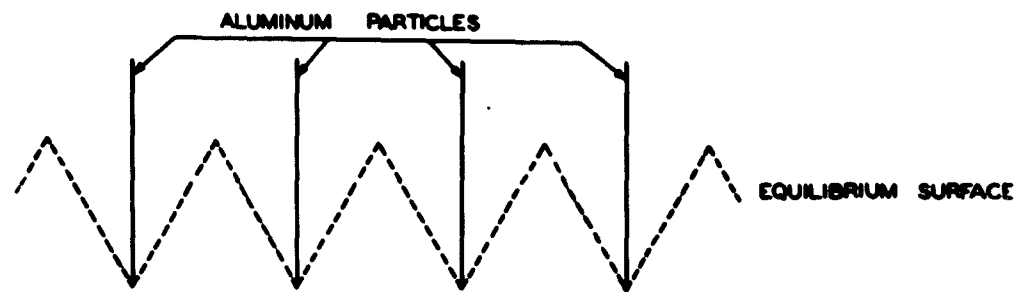Figure 4 - insert on page 17.
Insert new page 18.
Figure 8 - insert on page 19

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Fig. 4 Schematic drawing of equilibrium surface.



Fig. 8 Reduced burning rate versus total staple length at 2000 psia.
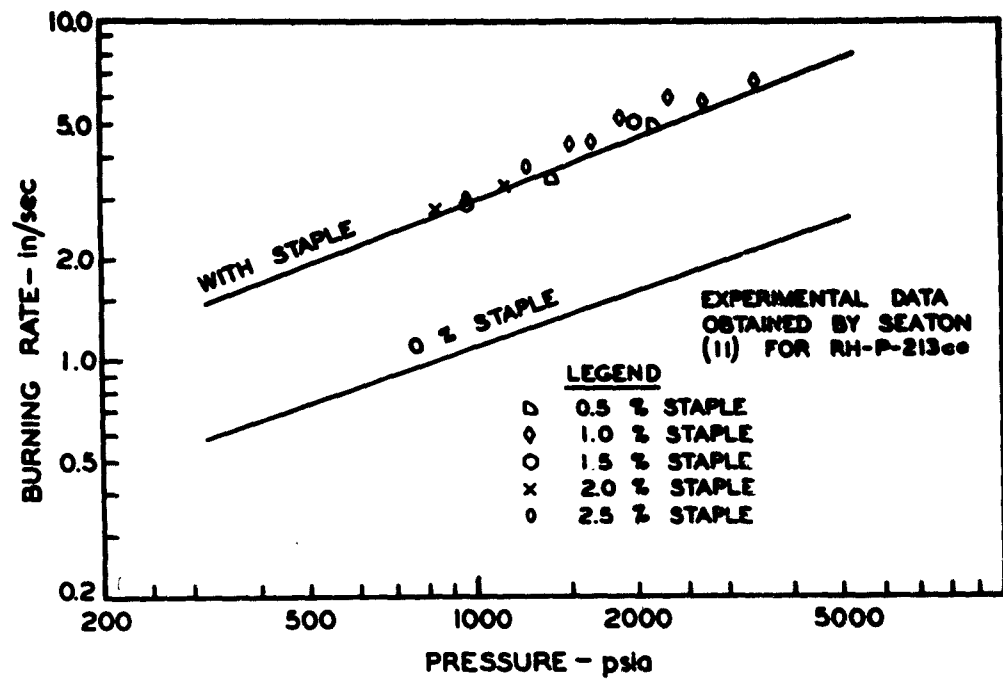
Fig. 5 P-r plot for varying staple concentrations .
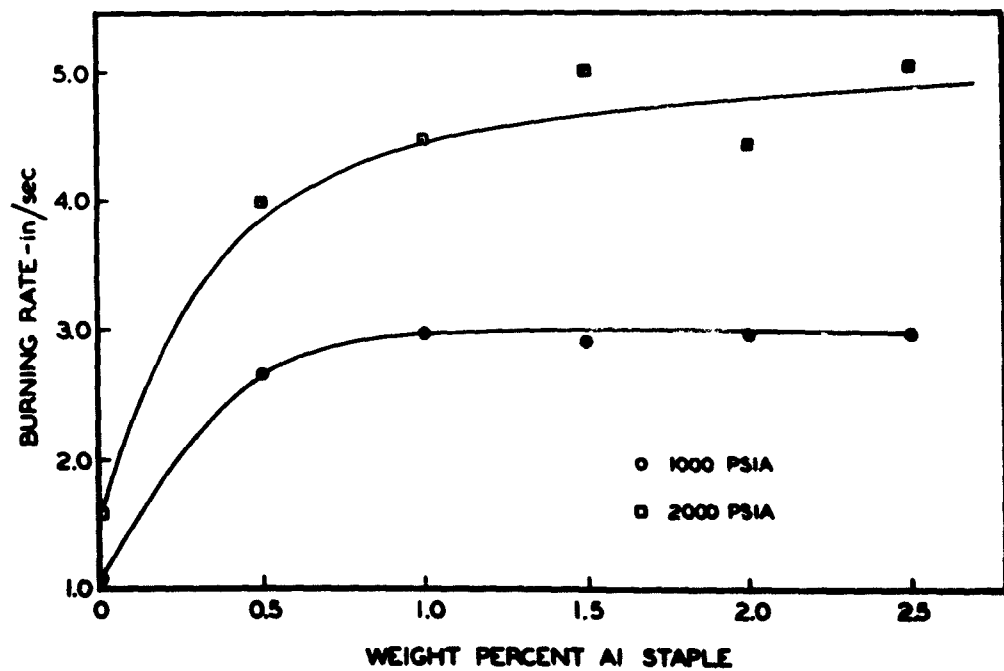
Fig. 6 Burning rate vs. weight percent Al staple for RH-P-213.

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## THE STUDY OF THE MECHANISM OF ENHANCED
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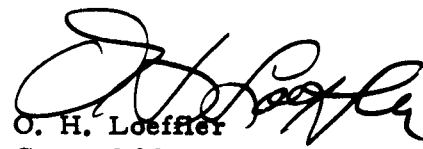
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March 20, 1964

Work reported herein was carried
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# FOREWORD

This report presents the text of a paper, presented at the American Institute of Aeronautics and Astronautics Solid Propellant Rocket Conference held at Palo Alto, California, January 29-31, 1964. A mathematical and geometrical analysis of burning of staple-containing solid propellants is presented which allows burning rate predictions with a minimum of information.

The present work is a more detailed treatment of the work presented in Rohm & Haas Quarterly Progress Report on Interior Ballistics, No. P-63-15, March, 1964.

## ABSTRACT

In this paper, the problem of burning-rate enhancement of solid propellants by incorporating metallic wires into the propellant matrix is treated by utilizing heat-transfer theory and geometry. It was found that heat conduction considerations incorporating the moving-heat-source restriction to the Fourier equation would adequately describe the process of "coning" and that the geometrical considerations of the coned surface together with the P-K-$\dot{r}$ data for the matrix propellant would give excellent predictions of the burning rate enhancement. The effects of wire length, wire concentration, and increased burning-rate pressure exponents due to the addition of wire are discussed. Experimental and theoretical P-$\dot{r}$ plots are presented for a typical staple-containing composite double-base propellant.

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## TABLE OF CONTENTS

THE STUDY OF THE MECHANISM OF ENHANCED
BURNING RATE OF SOLID PROPELLANTS

## Introduction

In 1944, Wilfong and Daniels (13) observed an increase in
burning rate of a solid propellant while attempting to measure the surface
temperature of a burning propellant with small thermocouples. The
increased burning rate was attributed to coning along the axial length of
the thermocouple wires. The possibility of utilizing this phenomenon

for increasing the effective linear burning rate of conventional solid propellants was investigated; much of the early work was carried on by the Atlantic Research Corporation (4, 8, 11) and others (1, 2, 5, 10). The earlier theoretical analysis of the phenomenon involved the simultaneous solution a multi-dimensional conduction-convection heat-transfer problem. However, lack of detailed information concerning boundary conditions and heat-transfer coefficients caused the calculated results to be somewhat erratic.

The current interest in high-burning-rate solid propellants for rocket motors with high thrust-short burning time requirements has led to a study of the effect of incorporating small metal wires in solid propellants. Substantial increases in rate have been achieved. A theoretical heat transfer analysis was combined with experimentally determined boundary conditions to describe the mechanism of rate increase and to provide guides for further experimental work. The results are presented in a straightforward and logical form to allow designers to predict burning-rate increases from the pressure-$K$-burning rate curves of standard propellants. The several parameters which require additional study are pointed out in the discussion.

## Analysis

Examination of the equation for the mass generation rate of a solid propellant

$$\dot{m} = \rho S \dot{r} \tag{1}$$

shows that the mass discharge rate $\dot{m}$ can be increased by

(1) increasing the burning rate, $\dot{r}$

(2) increasing the surface area, $S$

(3) increasing the burning rate and surface area simultaneously

where density, $\rho$, is constant.

To determine which of the three mechanisms prevails requires an agreement of motor firing with predictions based on the chosen mechanism.

Photographic studies (6, 7) of the combustion of solid propellants containing metal wires demonstrate that the "burning rate" of the propellant in close proximity to the wire exceeds the burning rate of the matrix and that the burning surface around the wires assumes the shape of a hollow, right-circular cone (5, 10). For the case of aluminum wire, the wire is heated to a high temperature at, or near, the propellant surface, either melts or ignites, and is subsequently ejected out into the main stream of gas flow. At present it is not possible to determine the exact mechanism or location of this "breakaway."

Based on these data and observations, it can be stated that the primary mass-generation-rate enhancement mechanism was that of increased surface due to coning along the wire. The following steps are postulated to describe the physical process of steady-state burning (see Fig. 1).

(1) Heat is convected from the high-temperature flame zone to the end of the wire which protrudes from the propellant.

(2) Heat is conducted down the metallic wire to the subsurface portion of the propellant.

(3) Heat is transferred to the propellant, initiating the decomposition reaction. This loss is neglected in the heat transfer analysis presented below to simplify the calculation, which is intended to demonstrate the availability of heat flux.

(4) The source of heat is moving with a velocity dictated by the physical properties of the system in a direction perpendicular to the burning surface.

On the basis of the model, the heat-transfer phenomena occurring in the staple itself can be approximated by transforming the coordinates of the Fourier equation to a system incorporating a moving

source of heat (8). For a system of fixed coordinates, the Fourier equation is

$$\frac{\partial T}{\partial t} = \alpha \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \tag{2}$$

where

$T$ = temperature

$x, y, z$ = coordinate variables

$\alpha$ = thermal diffusivity

$t$ = time.

An examination of Fig. 2 indicates that x should be replaced by $(vt + \xi)$ where v is the velocity of the moving heat source. For wires with a large length-to-diameter ratio, the steady-state, one-dimensional equation is

$$\frac{\partial^2 T}{\partial \xi^2} = - \frac{v}{\alpha} \frac{\partial T}{\partial \xi} . \tag{3}$$

If $T_i$ is the initial temperature of the wire and $\theta$ is defined as

$$\theta = T - T_i \tag{4}$$

the solution of Equation (2) may be expressed as

$$\theta = \theta_{max} e^{-\frac{v}{\alpha}\xi} \tag{5}$$

where $\theta_{max}$ is defined as the maximum temperature excess occurring in the wire.

Since the heat flux is defined as

$$\dot{q}/A = -k\frac{dT}{d\xi} \tag{6}$$

$$\frac{dT}{d\xi} = \frac{d\theta}{d\xi}$$

$$\dot{q}/A = k\frac{v}{a}\theta_{max}e^{-\frac{v}{a}\xi} \tag{7}$$

The assumptions involved in Equation (7) are as follows:

(1) There are no heat losses from the wire to the propellant.

(2) The velocity of propagation of the heat source, v, is constant.

(3) The heat flow is one-dimensional.

(4) The physical properties are constant.

At this point it would be possible to combine the convective heat losses and gains from the flame zone to the wire, the wire to the disassociated gases, and the wire to the propellant, as well as the multi-dimensional conduction of heat in the solid propellant, into a set of differential equations for iterative solutions on high-speed computing devices. However, since a great many of the variables would have to be approximated, perhaps as much information can be gained from a consideration of Equations (5) and (7) by studying limiting cases which result from choosing reasonable values of the physical properties of the system.

The problem (steady-state case) now resolves itself into the determination of v and $\theta_{max}$ for an aluminum wire or staple in a propellant matrix. The velocity of propagation of the heat source down the staple can be uniquely determined from a knowledge of the cone angle formed along the staple by the increased burning rate in this neighborhood (Fig. 1). However, the cone angles formed appear to be a unique function of the physical properties of the metallic particle

and of the nature of the matrix propellants, but independent of the pressure. The cone half-angles measured from movies varied between 16.5° and 23°, indicating a velocity of propagation of 2.5 to 3.0 times that of the base propellant. By approximating the burning rate of RH-P-213ce as 1 in/sec at 1000 psia, the velocity of propagation of the heat source, v, will be approximately 2.5 in/sec. Since the temperature at the exposed end of the staple was the melting point of aluminum or 660°C the maximum temperature excess, $\theta_{max}$, was

$$\theta_{max} = 660°C - 20°C = 640 \text{ deg C} = 1150 \text{ deg F}$$

The thermal diffusivity of aluminum is 0.1465 in²/sec. From Equation (7)

$$\dot{q}/A = 59.8 \, exp(-17\xi) \quad \frac{BTU}{in^2\text{-sec}} \tag{8}$$

Fig. 3 presents the calculated heat flux profile of an aluminum wire and it may readily be seen that the heat flux is quite high at small values of $\xi$. If one takes the much-used criterion (3, 5) that a heat flux of 0.38 BTU/in²-sec is sufficient to overcome the kinetic energy of activation and initiate the combustion reaction, then the coning action along the wire is adequately described by Equation (8). It may be maintained that Equation (8) does not rigorously describe the heat flux from the wire to the propellant and, admittedly, this is true. However, if one considers a heat balance over a small control volume of wire and propellant in a generalized for as

(rate of heat flux in) - (rate of heat flux out) = rate of accumulation

then Equation (8) predicts the maximum flow of heat from the wire to the propellant. Since this maximum value is approximately two orders of

magnitude higher than the heat flux required for the initiation of combustion it seems entirely safe to assume that at least 0.38 BTU/in²-sec are supplied to the propellant very near to the propellant surface.

The transient thermal response of metallic wires embedded in a solid propellant also poses a problem of major concern. The three methods by which one may obtain a solution for the transient response time are:

(1) the solution of equation (1) utilizing finite difference techniques and a high-speed computer,

(2) the analytical solution of the one-dimensional unsteady-state case of equation (1),

(3) the piecewise solution of the one-dimensional unsteady-state Fourier equation without the moving heat source with reliance upon the micro-window bomb studies (6, 7) to supply a reasonable approximation to the piecewise lengths which should be considered.

The third method was chosen, and is developed here.

Consider the equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad \text{for } 0 < x < l \tag{9}$$

with the transformation

$$\theta = T - T_2 \tag{10}$$

where $T_2$ = maximum temperature at end of wire
and the boundary conditions

$$\theta = \theta_1, \text{ as } t \to 0^+ \qquad \text{for } 0 < x < l \tag{11}$$

$$\theta = 0, \text{ as } x \to 0^+ \qquad \text{for } 0 < t \tag{12}$$

$$\frac{d\theta}{dx} = 0, \text{ as } x \to l^- \qquad \text{for } 0 < t \tag{13}$$

The solution can be put in the form

$$\theta = e^{-ah^2t} (A \cos ht + B \sin ht) \qquad (14)$$

$$\theta_{x,\ t} = -\sum_{k=0}^{\infty} \frac{2\theta_1}{\pi(2k+1)} \left( \cos\left[(2k+1)\pi\right] \right) \left( \sin\left[(2k+1)\frac{\pi x}{2\ell}\right] \right) e^{-at(2k+1)^2\pi^2/4\ell^2}$$

$$(15)$$

The analytic solution (method 2) is indicated in Appendix A.

The results of the micro-window bomb studies (6) indicated that approximately 24 mils of wire length were involved in the physical process at any given time. If one utilizes Equation (15) to compute the time required to raise the temperature of the aluminum staple to a temperature that is sufficient for auto-ignition (ca. 200° C), the time required is 3 msec. This 3-msec delay time is negligible in comparison with other delays which are inherent in the system and which will be discussed later.

A model can now be established to describe the phenomena of the burning of solid propellants containing randomly oriented metallic particles. The most realistic approach to the problem involves the calculation of a new equilibrium surface formed by the coning along the particle. The major assumption involved in this calculation is that, on the average, with a random staple orientation, the burning surface appears as shown in Fig. 4. The formation of the burning surface (Fig. 4) permits calculation of the new equilibrium surface on the basis of the burning rate enhancement (or the cone angle) along the staple. Calculations normalized to one square inch of original surface area are summarized in Table I from which it is seen that the new equilibrium surface is a function which depends only on the sine of the cone angle or the ratio of the burning rate along

the staple to the burning rate of the matrix propellant, and not on the concentration of the staple in the propellant. This phenomenon is a unique property of the right-circular cone.

Table I

Effect of Aluminum Staple Concentration on Equilibrium Burning Surface

Cone Half-angle = 23°

| Weight % Staple | Staple Spacing Cubic Packing, mils | New Equilibrium Surface Area, in$^2$ |
|---|---|---|
| 2 | 27.4 | 2.56 |
| 1 | 38.6 | 2.56 |
| 0.5 | 55.0 | 2.56 |
| 0.02 | 160.0 | 2.56 |

The method for predicting burning rates now resolves itself into the calculation of a new K value based on the new equilibrium surface established by coning at the propellant surface to establish an effective or "pseudo" burning rate based on the propellant surface area without considering the cone areas. Expressed mathematically

$$\frac{K_2}{K_1} = \frac{1}{\sin\frac{\theta}{2}}$$

$$\frac{\dot{r}_2}{\dot{r}_1} = \left(\sin\frac{\theta}{2}\right)^{n/(n-1)}$$

where　$\theta$ = cone angle

Subscript $_2$ refers to staple propellant

Subscript $_1$ refers to staple free

n = pressure exponent of burning rate.

Fig. 5 presents a plot of burning rate versus pressure for the base propellant RH-P-213ce (see Appendix B for formulation), and for the same propellant containing aluminum staple in any amount suf-

ficient to establish stable burning. The data of Seaton (12) are also presented on the same plot in varying staple concentrations from 0.5% to 2.5%. The agreement is seen to be excellent.

## Discussion

### 1. Effect of Staple Concentration

The effect of staple concentration on burning rate theoretically should be negligible after a certain point but for practical purposes it has been found (12) that the burning rate increases slightly with increasing staple concentration. Fig. 6 presents a plot of burning rate versus staple composition for RH-P-213 at 1000 and 2000 psia (12). The explanation lies in the visualization of the randomly orientated staples in the propellant matrix. As a cone is formed along a staple, it is possible to begin the formation of new cones along the inner surface of the first cone, causing relatively large pieces of propellant to detach themselves from the main body of the propellant during combustion, thus raising the effective burning rate.

The most important effect of staple concentration is the effect on the time delay in the establishment of the new equilibrium surface. This delay time is the time required for the cones of each individual staple to mutually intersect with the cones formed by each of the nearest neighbors. For cubic packings, it is readily seen that the delay time is inversely proportional to the square root of the number density of staples in the propellant or, more specifically, the staple concentration. The delay time is calculated by (1) assuming a staple concentration for a given staple dimension, (2) assuming an orientation ($\frac{1}{3}$ of the total number of staples aligned parallel to each cartesian coordinate axis), (3) computing the staple spacing, and (4) determining the time required for the propellant to regress from one staple to a point midway between it and its nearest neighbor. For a 0.5 weight %

loading, this delay should be on the order of 30 msec.  For reference, staple spacing based on a cubic packing as a function of weight percent in RH-P-213ce is presented in Table I.

A spacing of 160 mils (for a staple 4.5 mils by 0.7 mils by 312 mils in a cubic lattice) corresponds to 0.02 weight % staple in the propellant matrix.  However, this calculation does not, and is not intended to, mean that 0.02 weight percent  is the optimum staple concentration to achieve the maximum increase in burning rate.  The time required to establish a new surface is an important parameter which is composition dependent.  It has been established by Seaton (12) that a minimum concentration of 0.5 weight % staple is necessary to establish stable burning in a 5-inch end-burning web.

## 2.    Effect of Staple Length

Staple length has two pronounced effects upon the burning characteristics of solid propellants — as the length of staple is decreased the minimum concentration of staple required to establish a steady-state burning surface is increased; and, as the length of staple is increased, the overall mass-discharge rate increases and passes through a maximum.

From the geometrical considerations previously presented the calculation of spacing or concentration for a staple of given length necessary to achieve stable burning is a relatively simple matter.  Table II presents the calculated minimum concentration of staple necessary to produce a steady-state coned equilibrium surface as a function of staple length.  Since the compositions presented in Table II represent

Table II

Minimum Staple Concentration Required to Establish
Equilibrium Burning Surface as a Function of Staple Length

| Staple Length, mils | Minimum Conc. wt. % |
|---|---|
| 312 | 0.025 |
| 200 | 0.06 |
| 100 | 0.24 |
| 50 | 0.95 |

minimum values, it is apparent that the effect of staple length on burning rate could be quite drastic with staples less than 100 mils in length. This has indeed been demonstrated experimentally by Brown and Thompson (6, 7).

The effect of staple length on burning rate enhancement has been examined utilizing the following procedure:

(1) Assume a weight concentration of staple in a given propellant such as RH-P-213ce.

(2) Assume a staple length and calculate a staple number density.

(3) From the number density (assuming $\frac{1}{3}$ of the total number of staples are aligned perpendicular to the burning surface), compute the staple spacing on a cubic matrix.

(4) From the previous data compute the time required for the flame front to proceed down the staple and through the propellant to the next staple.

(5) Compute the overall burning rate.

The results of a series of these calculations for aluminum staple in RH-P-213ce are presented in Figs. 7 and 8 for pressures of 1000 and 2000 psia. The burning rates are normalized to the burning rate computed for a staple length of 312 mils. The burning rate passes through a maximum at a staple length of approximately 5000 mils.

### 3. Effect of Staple on Pressure Exponent

From the results of experimental firings, it has been found that the addition of metallic staple to plastisol-type solid propellants increases the pressure exponent of the burning-rate equation by as much as 50% over the entire pressure range investigated (1000-3000 psia). This phenomenon is postulated to result from a decrease in the distance between the flame zone and the propellant surface with increasing pressure. In effect, this decrease in reaction-zone thickness has the effect of decreasing the heat loss from the staple to the oxygen-rich decomposition products leaving the propellant surface, thus providing a larger quantity of available

energy to initiate propellant decomposition in the subsurface area. This could have the qualitative effect of increasing the pressure exponent of the burning rate above the value obtained for the matrix propellant.

### 4. Effect of Staple Size on Burning Rate

Data (6, 7) on specially prepared samples of staple containing propellants incorporating staples of widely varying dimensions have revealed that the actual cross-section of the metallic element has no effect on the burning rate for pressures up to 1750 psi. This is in direct contradiction to other mathematical analyses,yet the statement is in agreement with experiment data. Logically one should expect little or no effect of cross-section on burning rate as long as the wires under consideration exhibit a large length-to-diameter ratio such that the temperature profile can be determined by a one-dimensional analysis.

### 5. Problems Involved in a Rigorous Analysis

The problems involved in a complete heat-transfer analysis were summarized in the introduction—a lack of knowledge concerning the detailed boundary conditions. As examples one should be able to pinpoint the heat-transfer coefficients between the flame zone and the wire as well as the coefficient between the subsurface portion of the wire and the propellant. A knowledge of all interfacial temperatures as well as the temperature gradient in the dissociated gases issuing from the surface is also imperative. In addition to the above, the heat generated by the kinetic decomposition of the propellant would need to be specified as well as its distribution to various parts of the system.

An even more intriguing problem is presented by a theoretical analysis describing the actual orientation of the staple in the matrix propellant. A solution to this problem would indeed be an extension to the problem of the multi-dimensional random walk.

This problem has solutions for specialized cases concerning the length of each displacement after a large number of steps but for only one particle at a time. The case for many particles with a small number of displacements has not, to the author's knowledge, been treated theoretically.

## Conclusions

A model has been proposed which adequately describes the mechanism of combustion of solid propellants which contain metallic wire or staple and is in complete agreement with the experimental information which is available to date. The mechanism of burning rate enhancement has been shown to be dependent on heat conduction and may be described by the well-known Fourier equation incorporating a moving heat source. The inclusion of geometrical considerations makes possible the prediction of burning rate enhancement from the $P-K-\dot{r}$ curves of the matrix propellant with a knowledge only of the cone angle formed along the wire during burning.

## ACKNOWLEDGMENTS

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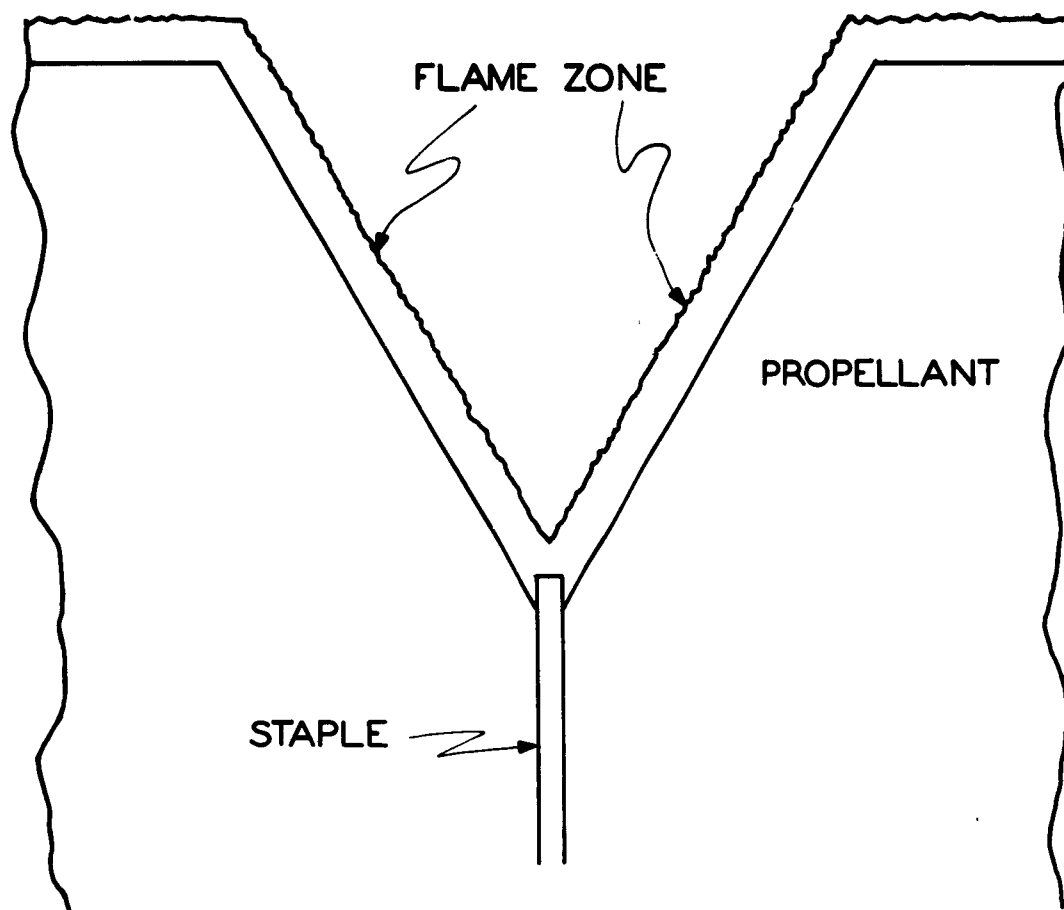
Fig. I Combustion model .
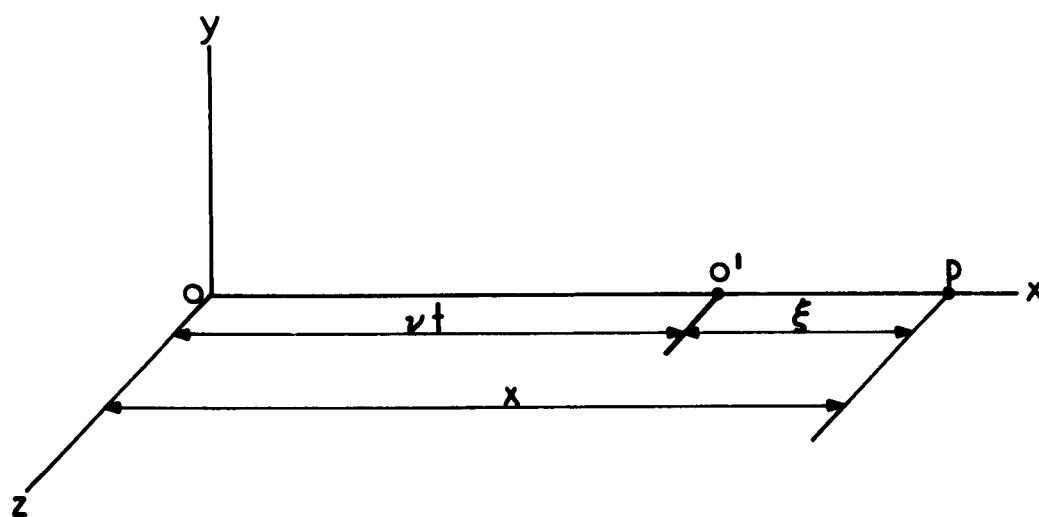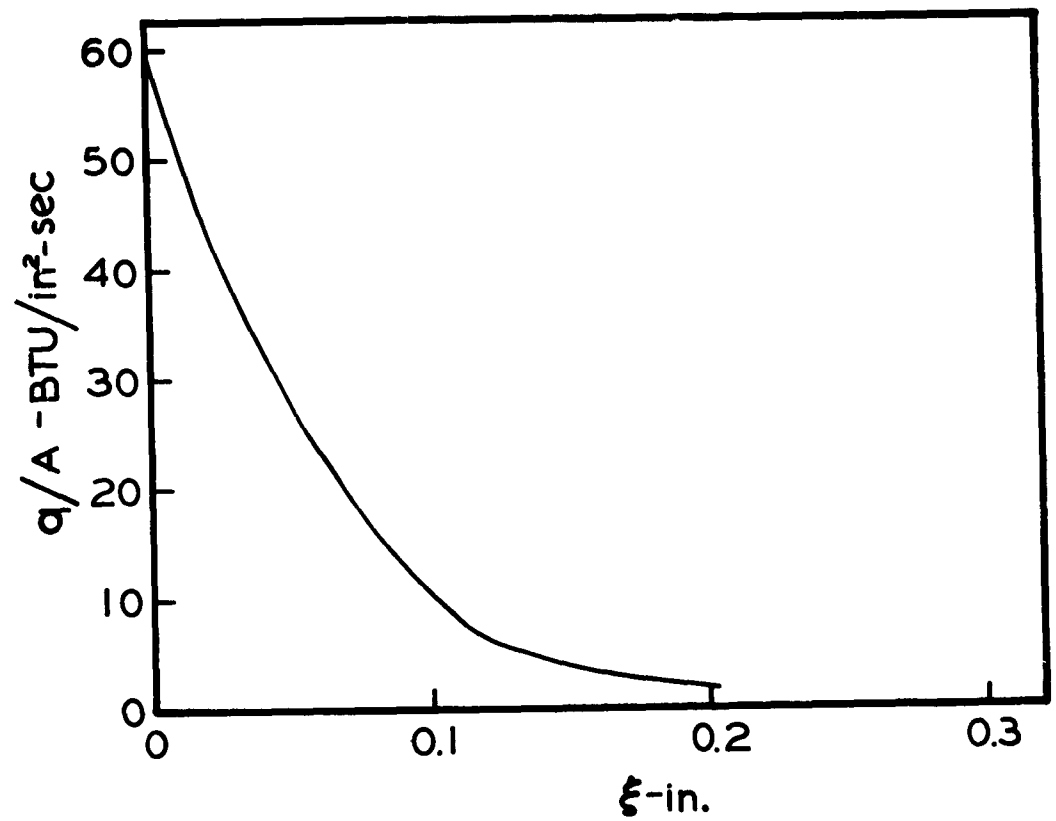


Fig. 2 Coordinate system for moving heat source .

Fig. 3 Heat flux vs. distance from heat source.

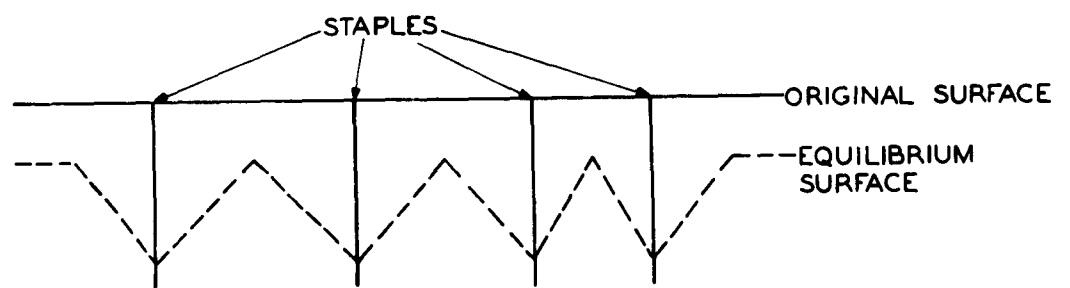

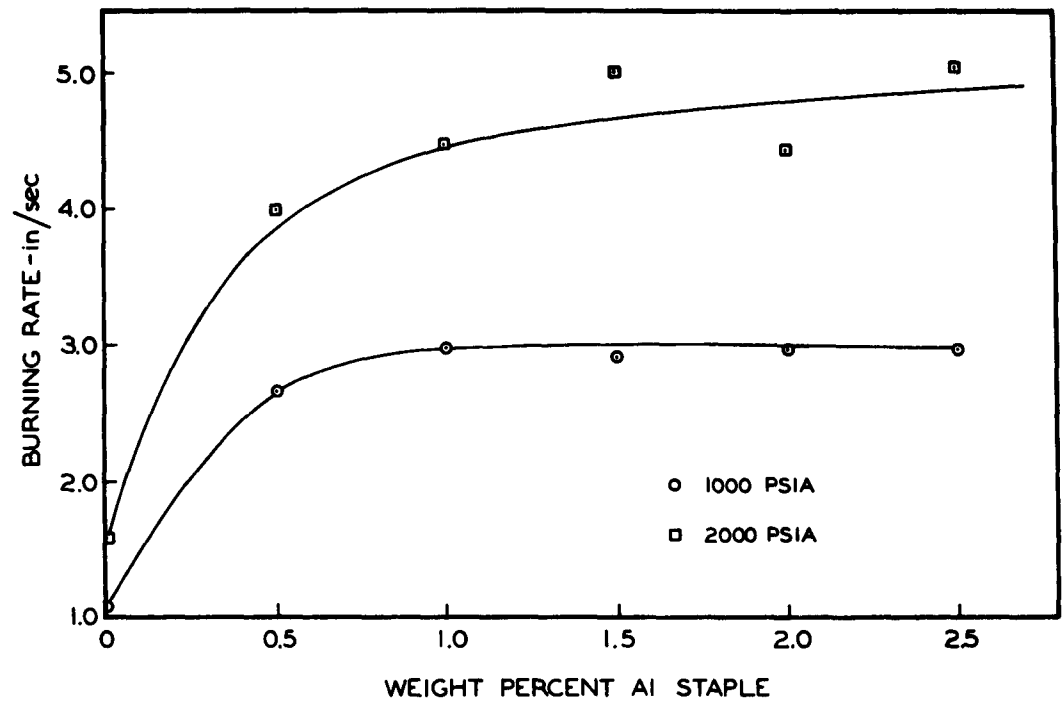Fig. 4 Schematic drawing of equilibrium surface.

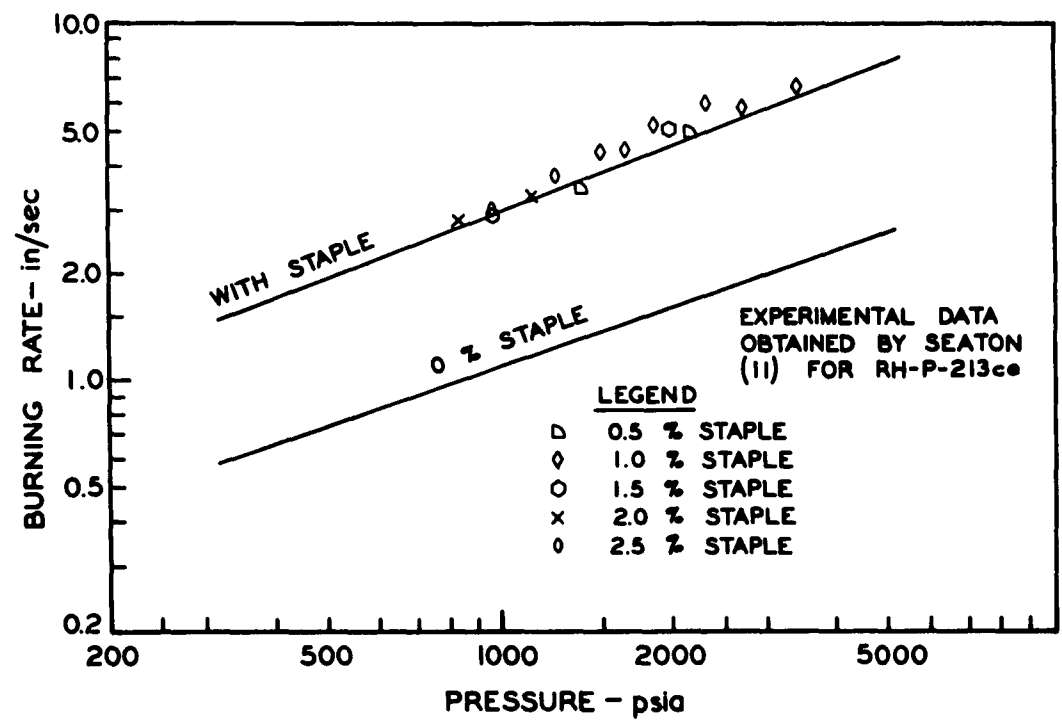Fig. 5 P-r plot for varying staple concentrations .



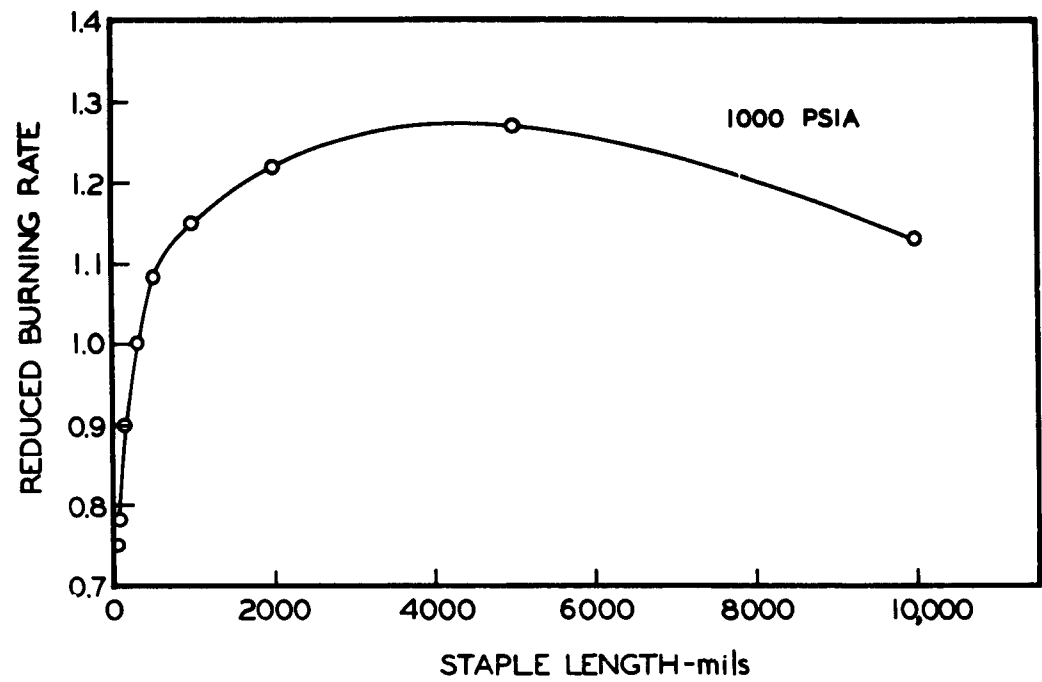Fig. 6 Burning rate vs. weight percent Al staple for RH-P-213.

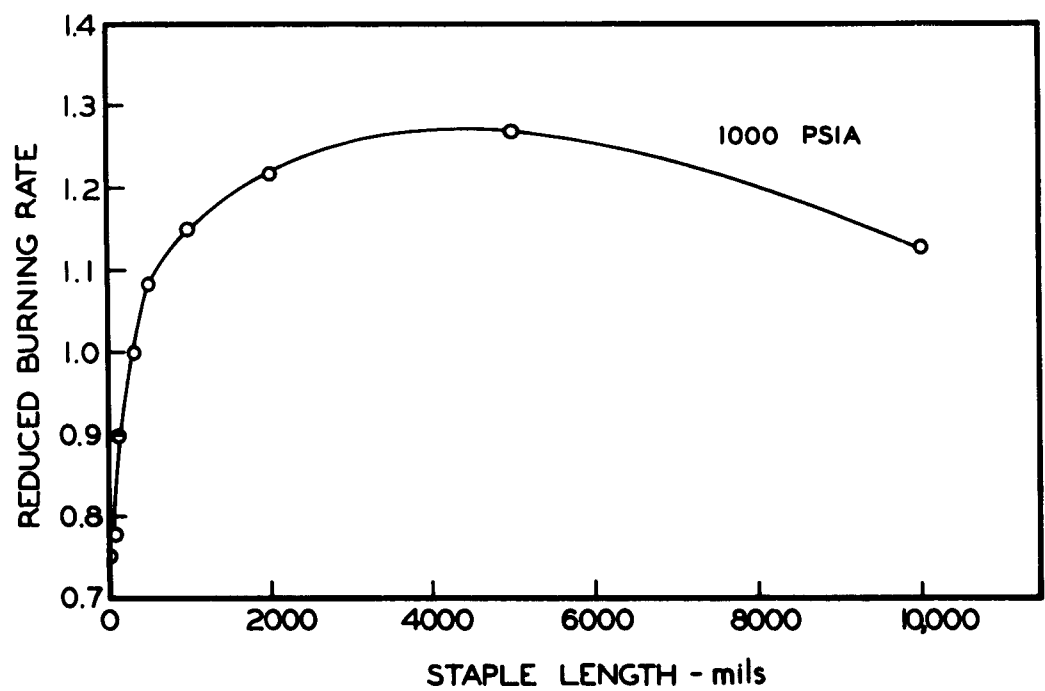Fig. 7  Reduced burning rate versus total staple length at 1000 psia .



Fig. 8  Reduced burning rate versus total staple length at 2000 psia .

## Appendix A

If one considers the unsteady-state Fourier equation incorporating a moving heat source:

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{v}{h^2} \frac{\partial \phi}{\partial \xi} = \frac{1}{h^2} \frac{\partial \phi}{\partial t} \qquad (A-1)$$

$$\phi = T - T_2$$

$$T_2 = T_{max}$$

a solution (at constant v) may be written in the form

$$\phi = \psi(\xi) \tau(t) \qquad (A-2)$$

Utilizing standard techniques

$$\frac{\partial^2 \phi}{\partial \xi^2} + \beta \frac{\partial \phi}{\partial \xi} = \frac{1}{h^2} \frac{\partial \phi}{\partial t} = -a^2 \qquad (A-3)$$

$$\beta = \frac{v}{h^2}$$

A solution may now be expressed as

$$\phi = e^{-a^2 h^2 t} [B (\cosh v\xi + \sinh \alpha\xi)(\cosh \beta'\xi - \sinh \beta'\xi) +$$

$$+ C (\cosh \alpha\xi - \sinh \alpha\xi)(\cosh \beta'\xi - \sinh \beta'\xi)] \qquad (A-4)$$

where

$$\alpha = \tfrac{1}{2} \sqrt{\beta^2 - 4a^2}$$

$$\beta' = \beta/2$$

a, B, C are arbitrary constants.

The boundary conditions are defined as:

$$\phi = \phi_1 \qquad\qquad 0 < x < \ell \qquad\qquad \text{as } t \to 0^+ \qquad (A-5)$$

$$\phi = 0 \qquad\qquad 0 < t \qquad\qquad \text{as } \xi \to 0^+ \qquad (A \cdot 6)$$

$$\frac{d\phi}{d\xi} = 0 \qquad\qquad 0 < t \qquad\qquad \text{as } \xi \to \ell^- \qquad (A-7)$$

· From Equations (A-5), (A-6), and (A-7)

$$B = \frac{\phi_1}{\Omega_2 - \Lambda\Omega_1}$$

$$C = \frac{-\phi_1\Lambda}{\Omega_2 + \Lambda\Omega_2}$$

where

$$\Omega_1 \;=\; (\cosh \alpha\xi - \sinh \alpha\xi)(\cosh \beta'\xi - \sinh \beta'\xi)$$

$$\Omega_2 \;=\; (\cosh \alpha\xi + \sinh \alpha\xi)(\cosh \beta'\xi - \sinh \beta'\xi)$$

$$\Lambda \;=\; \frac{\begin{bmatrix}\sinh \alpha\ell\end{bmatrix}[\alpha(\cosh \beta'\ell - \sinh \beta'\ell) + \beta'(\sinh \beta'\ell - \cosh \beta'\ell)] + \begin{bmatrix}\cosh \alpha\ell\end{bmatrix}[\alpha(\cosh \beta'\ell - \sinh \beta'\ell) + \beta'(\sinh \beta'\ell - \cosh \beta'\ell)]}{\begin{bmatrix}\sinh \beta'\ell\end{bmatrix}[\beta'(\cosh \alpha\ell - \sinh \alpha\ell) - \alpha(\cosh \alpha\ell - \sinh \alpha\ell)] + \begin{bmatrix}\cosh \beta'\ell\end{bmatrix}[\alpha(\sinh \alpha\ell - \cosh \alpha\ell) + \beta'(\sinh \alpha\ell - \cosh \alpha\ell)]}$$

thus

$$\phi \;=\; \phi_1 e^{-a^2 h^2 t}\left[\frac{\Omega_2}{\Omega_2 - \Lambda\Omega_1} - \frac{\Lambda\Omega_1}{\Omega_2 + \Lambda\Omega_1}\right]$$

The constant a, and therefore $\alpha$ and $\beta'$ are now determined from Equation (A-6). Thus a solution of the form

$$\phi = \phi_1 e^{-h^2 t} g(v, \ell, \phi_1, \xi) \left[ \frac{\Omega_2}{\Omega_2 - \Lambda\Omega_1} = \frac{\Lambda\Omega_1}{\Omega_2 + \Lambda\Omega_1} \right]$$

may be written for the unsteady-state temperature distribution in a metallic staple.

Appendix B

Composition of RH-P-213

| Component | Wt. % |
|---|---|
| Double-Base Powder | 12.00 |
| Triethylene Glycol Dinitrate | 38.00 |
| Ammonium Perchlorate | 45.00 |
| Aluminum | 4.00 |
| Resorcinol | 1.00 |

Staple Dimensions        4.5×0.7×312 mils
Prop. density = 0.06 lbm/in$^3$

Initial distribution of this report was made
in accordance with the Joint Army-Navy-
Air Force mailing lists for Solid Propellant
and Liquid Propellant technical information
plus approved supplements